

Mathematics — Solved Paper 2014

SECTION A (40 Marks)

(Answer *all* questions from this Section)

Question 1 :

- (a) Ranbir borrows ₹ 20,000 at 12% per annum compound interest. If he repays ₹ 8,400 at the end of the first year and ₹ 9,680 at the end of the second year, find the amount of loan outstanding at the beginning of the third year. [3]

Solution :

For 1st year : P = ₹ 20,000; R = 12% and T = 1 year

$$\therefore \text{Interest} = \frac{\text{₹ } 20,000 \times 12 \times 1}{100} = \text{₹ } 2,400$$

$$\begin{aligned} \text{Amount} &= P + I \\ &= \text{₹ } 20,000 + \text{₹ } 2,400 = \text{₹ } 22,400 \end{aligned}$$

$$\therefore \text{Paid at the end of 1st year} = \text{₹ } 8,400$$

$$\begin{aligned} \therefore \text{Loan outstanding at the beginning of the 2nd year} \\ &= \text{₹ } 22,400 - \text{₹ } 8,400 = \text{₹ } 14,000 \end{aligned}$$

For 2nd year : P = ₹ 14,000; R = 12% and T = 1 year

$$\therefore \text{Interest} = \frac{\text{₹ } 14,000 \times 12 \times 1}{100} = \text{₹ } 1,680$$

$$\begin{aligned} \text{Amount} &= P + I \\ &= \text{₹ } 14,000 + \text{₹ } 1,680 = \text{₹ } 15,680 \end{aligned}$$

$$\therefore \text{Paid at the end of 2nd year} = \text{₹ } 9,680$$

$$\begin{aligned} \therefore \text{Loan outstanding at the beginning of the 3rd year} \\ &= \text{₹ } 15,680 - \text{₹ } 9,680 = \text{₹ } 6,000 \end{aligned}$$

Ans.

- (b) Find the values of x , which satisfy the inequation $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2$, $x \in W$.

Graph the solution set on the number line.

[3]

Solution :

$$\Rightarrow -\frac{17}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2$$

$$\Rightarrow -17 < 3 - 4x \leq 12 \quad [\text{Multiplying by L.C.M. of 6, 2 and 3}]$$

$$\Rightarrow -17 < 3 - 4x \quad \text{and} \quad 3 - 4x \leq 12$$

$$\Rightarrow 4x < 20 \quad \text{and} \quad 4x \geq -9$$

$$\Rightarrow x < 5 \quad \text{and} \quad x \geq -2\frac{1}{4}$$

$$\Rightarrow -2\frac{1}{4} \leq x < 5, x \in W$$

$\therefore$ Values of x are : 0, 1, 2, 3 and 4.

Ans.

Required number line :



Ans.

(c) A die has 6 faces marked by the given numbers as shown below :

$$\boxed{1} \quad \boxed{2} \quad \boxed{3} \quad \boxed{-1} \quad \boxed{-2} \quad \boxed{-3}$$

The die is thrown once. What is the probability of getting

(i) a positive integer.

(ii) an integer greater than -3 .

(iii) the smallest integer. [4]

Solution :

Total number of possible outcomes = 6

(i) $\therefore$ Favourable outcomes = 3 [1, 2 and 3]

$$\therefore \text{Required probability} = \frac{3}{6} = \frac{1}{2} \quad \text{Ans.}$$

(ii) $\therefore$ Favourable outcomes = 5 [1, 2, 3, -1 and -2]

$$\therefore \text{Required probability} = \frac{5}{6} \quad \text{Ans.}$$

(iii) $\therefore$ Favourable outcomes = 1 [*i.e.* -3]

$$\therefore \text{Required probability} = \frac{1}{6} \quad \text{Ans.}$$

Question 2 :

$$(a) \text{ Find } x, y \text{ if } \begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} y \\ 3 \end{bmatrix}. \quad [3]$$

Solution :

$$\Rightarrow \begin{bmatrix} 2+0 \\ -3+2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-6 \\ -3+2x+3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix} \Rightarrow 2y = -4 \text{ and } 2x = 6$$

$$\therefore x = 3 \text{ and } y = -2 \quad \text{Ans.}$$

(b) Shahrukh opened a Recurring Deposit Account in a bank and deposited ₹ 800

per month for $1\frac{1}{2}$ years. If he received ₹ 15,084 at the time of maturity, find

the rate of interest per annum. [3]

Solution :

Given : Monthly deposit (P) = ₹ 800, $n = \frac{3}{2} \times 12$ months = 18 months
and maturity value (M.V.) = ₹ 15,084

$$\therefore \text{M.V.} = P \times n + P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow ₹ 15,084 = 800 \times 18 + \frac{800 \times 18 \times 19}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow ₹ 15,084 = 14,400 + 114r$$

$$\Rightarrow r = 6\% \quad \text{Ans.}$$

- (c) Calculate the ratio in which the line joining A(-4, 2) and B(3, 6) is divided by point P(x, 3). Also, find (i) x (ii) Length of AP. [4]

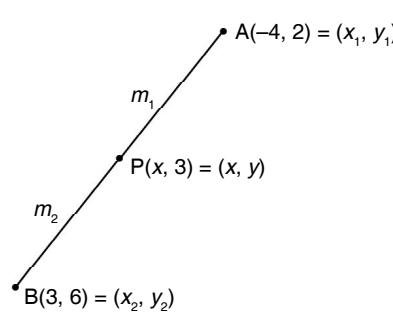
Solution :

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow 3 = \frac{m_1 \times 6 + m_2 \times 2}{m_1 + m_2}$$

$$\Rightarrow 3m_1 + 3m_2 = 6m_1 + 2m_2$$

$$\Rightarrow 3m_1 = m_2$$

$$i.e. \quad \frac{m_1}{m_2} = \frac{1}{3}$$


$\therefore$ Required ratio = 1 : 3 Ans.

(i) $x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$$\Rightarrow x = \frac{1 \times 3 + 3 \times -4}{1 + 3} = \frac{-9}{4}$$
Ans.

(ii) $AP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let A = (-4, 2) = (x₁, y₁)

and P = (x, 3) = (-9/4, 3) = (x₂, y₂)

$$= \sqrt{\left(-\frac{9}{4} + 4\right)^2 + (3 - 2)^2}$$

$$= \sqrt{\left(\frac{7}{4}\right)^2 + 1} = \sqrt{\frac{65}{16}} = \frac{1}{4}\sqrt{65}$$
Ans.

Question 3 :

- (a) Without using trigonometric tables, evaluate

$$\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ. \quad [3]$$

Solution :

$$= \sin^2 34^\circ + \sin^2 (90^\circ - 34^\circ) + 2 \tan 18^\circ \tan (90^\circ - 18^\circ) - (\sqrt{3})^2$$

$$= \sin^2 34^\circ + \cos^2 34^\circ + 2 \tan 18^\circ \cot 18^\circ - 3$$

$$= 1 + 2 \tan 18^\circ \times \frac{1}{\tan 18^\circ} - 3 = 1 + 2 - 3 = 0$$
Ans.

- (b) Using the remainder and factor theorem, factorise the following polynomial $x^3 + 10x^2 - 37x + 26$. [3]

Solution :

$$\text{For } x = 1, \text{ the given expression} = (1)^3 + 10(1)^2 - 37(1) + 26$$

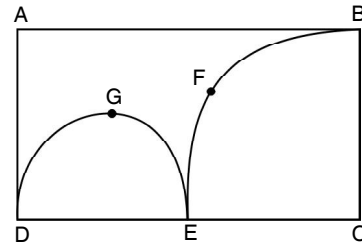
$$= 1 + 10 - 37 + 26 = 0$$

$\therefore (x - 1)$ is a factor of the given expression

$$\begin{aligned}
&\therefore x^3 + 10x^2 - 37x + 26 \\
&= (x - 1) (x^2 + 11x - 26) \\
&= (x - 1) (x^2 + 13x - 2x - 26) \\
&= (x - 1) [x (x + 13) - 2 (x + 13)] \\
&= (x - 1) (x + 13) (x - 2) \quad \text{Ans.}
\end{aligned}$$

$$\begin{array}{r}
x^2 + 11x - 26 \\
x - 1 \overline{) x^3 + 10x^2 - 37x + 26} \left(\begin{array}{r} x^3 - x^2 \\ - \quad + \\ \hline 11x^2 - 37x \\ 11x^2 - 11x \\ - \quad + \\ \hline -26x + 26 \\ -26x + 26 \\ + \quad - \\ \hline \times \end{array} \right.
\end{array}$$

- (c) In the given figure, ABCD is a rectangle. AB = 14 cm, BC = 7 cm. From the rectangle, a quarter circle BFEC and a semicircle DGE are removed. Calculate the area of the remaining piece of the rectangle. (Take $\pi = \frac{22}{7}$) [4]



Solution :

Radius of the quarter circle BFEC = BC = 7 cm i.e. R = 7 cm

EC = BC = 7 cm $\Rightarrow$ DE = DC - EC = AB - EC = 14 cm - 7 cm = 7 cm

$\Rightarrow$ Radius (r) of semicircle DGE = $\frac{7}{2}$ cm

$\therefore$ The area of the remaining piece of the rectangle

= Area of rectangle ABCD - Area of quarter circle BFEC - Area of semicircle DGE

$$= AB \times BC - \frac{1}{4} \pi R^2 - \frac{1}{2} \pi r^2$$

$$= \left[14 \times 7 - \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 - \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right] \text{cm}^2$$

$$= 40.25 \text{ cm}^2$$

Ans.

Question 4 :

- (a) The numbers 6, 8, 10, 12, 13 and x are arranged in an ascending order. If the mean of the observations is equal to the median, find the value of x. [3]

Solution :

Given that the numbers are in ascending order

and, mean = median

$$\Rightarrow \frac{6+8+10+12+13+x}{6} = \frac{1}{2} \left[\left(\frac{6}{2} \right)^{\text{th}} \text{ term} + \left(\frac{6}{2} + 1 \right)^{\text{th}} \text{ term} \right] \quad [\because n = 6]$$

$$\Rightarrow \frac{49+x}{6} = \frac{1}{2}[3^{\text{rd}} \text{ term} + 4^{\text{th}} \text{ term}]$$

$$\Rightarrow \frac{49+x}{6} = \frac{1}{2}[10 + 12] \Rightarrow x = 17$$

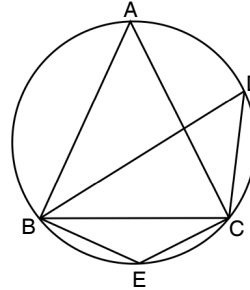
Ans.

(b) In the figure, $\angle DBC = 58^\circ$. BD is a diameter of the circle. Calculate :

(i) $\angle BDC$

(ii) $\angle BEC$

(iii) $\angle BAC$



[3]

Solution :

(i) BD is a diameter

$$\Rightarrow \angle BCD = 90^\circ \quad [\text{angle of semicircle}]$$

$$\Rightarrow \angle DBC + \angle BDC = 90^\circ$$

$$\Rightarrow 58^\circ + \angle BDC = 90^\circ$$

$$\text{i.e.} \quad \angle BDC = 32^\circ \quad \text{Ans.}$$

(i) Since, opposite angles of a cyclic quadrilateral are supplementary, therefore in cyclic quadrilateral BECD,

$$\angle BEC + \angle BDC = 180^\circ$$

$$\Rightarrow \angle BEC + 32^\circ = 180^\circ$$

$$\text{i.e.} \quad \angle BEC = 148^\circ \quad \text{Ans.}$$

(iii) In cyclic quadrilateral ABEC,

$$\Rightarrow \angle BAC + \angle BEC = 180^\circ$$

$$\Rightarrow \angle BAC + 148^\circ = 180^\circ$$

$$\text{i.e.} \quad \angle BAC = 32^\circ \quad \text{Ans.}$$

OR Since, angles of same segment are equal,

$$\therefore \angle BAC = \angle BDC = 32^\circ \quad \text{Ans.}$$

(c) Use graph paper to answer the following questions. (Take 2 cm = 1 unit on both axis)

(i) Plot the points A (−4, 2) and B(2, 4).

(ii) A' is the image of A when reflected in the y-axis. Plot it on the graph paper and write the coordinates of A'.

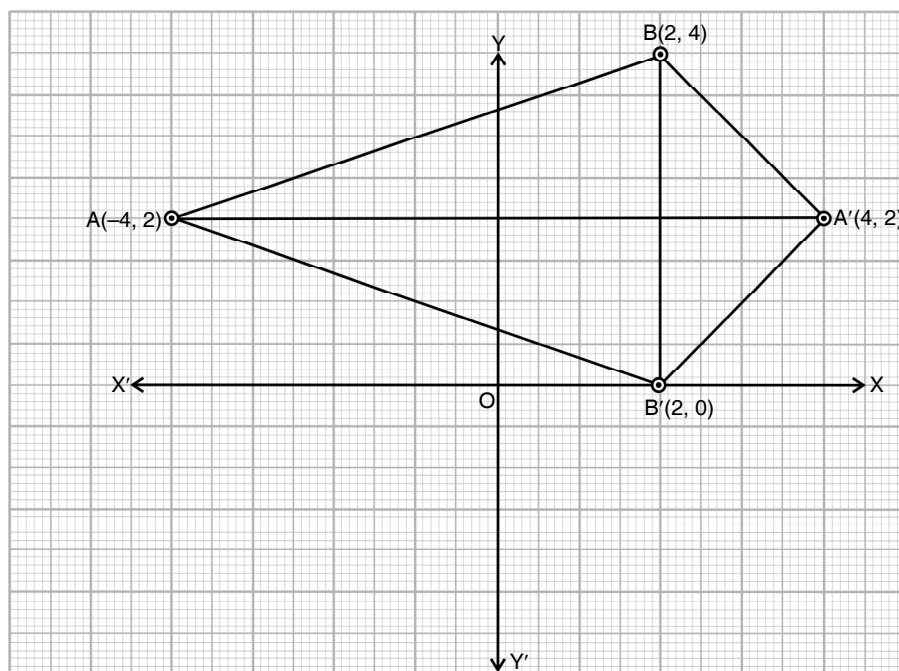
(iii) B' is the image of B when reflected in the line AA'. Write the coordinates of B'.

(iv) Write the geometric name of the figure ABA'B'.

(v) Name a line of symmetry of the figure formed. [4]

Solution :

(i)



(ii) $A' = (4, 2)$.

Ans.

(iii) $B' = (2, 0)$.

Ans.

(iv) Kite-shaped quadrilateral.

Ans.

(v) AA'

Ans.

SECTION B (40 Marks)

(Answer any **four** questions from this Section)

Question 5 :

(a) A shopkeeper bought a washing machine at a discount of 20% from a wholesaler, the printed price of the washing machine being ₹ 18,000. The shopkeeper sells it to a consumer at a discount of 10% on the printed price. If the rate of sales tax is 8%, find :

(i) the VAT paid by the shopkeeper.

(ii) the total amount that the consumer pays for the washing machine. [3]

Solution :

(i) **For the shopkeeper :**

$$\text{C.P.} = ₹ 18,000 - 20\% \text{ of } ₹ 18,000 = ₹ 14,400$$

$$\text{S.P.} = ₹ 18,000 - 10\% \text{ of } ₹ 18,000 = ₹ 16,200$$

The VAT = Sales tax on (S.P. – C.P.)

$$= 8\% \text{ of } (₹ 16,200 - ₹ 14,400) = ₹ 144$$

Ans.

(ii) **The total amount that the consumer pays**

$$= ₹ 16,200 + 8\% \text{ of } ₹ 16,200 = ₹ 17,496$$

Ans.

(b) If $\frac{x^2 + y^2}{x^2 - y^2} = \frac{17}{8}$, then find the value of (i) $x : y$ (ii) $\frac{x^3 + y^3}{x^3 - y^3}$. [3]

Solution :

(i) On applying componendo and dividendo, we get :

$$\frac{x^2+y^2+x^2-y^2}{x^2+y^2-x^2+y^2} = \frac{17+8}{17-8} \quad \text{i.e.} \quad \frac{2x^2}{2y^2} = \frac{25}{9}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{25}{9} \quad \text{i.e.} \quad \frac{x}{y} = \frac{5}{3} \quad \Rightarrow x : y = 5 : 3 \quad \text{Ans.}$$

Alternative method :

$$\frac{x^2+y^2}{x^2-y^2} = \frac{17}{8} \Rightarrow 17x^2 - 17y^2 = 8x^2 + 8y^2$$

$$\text{i.e. } 9x^2 = 25y^2 \Rightarrow \frac{x^2}{y^2} = \frac{25}{9} \quad \text{and} \quad \frac{x}{y} = \frac{5}{3} \quad \text{i.e. } x : y = 5 : 3 \quad \text{Ans.}$$

$$(ii) \quad \frac{x}{y} = \frac{5}{3} \Rightarrow \frac{x^3}{y^3} = \frac{125}{27}$$

$$\text{i.e.} \quad \frac{x^3+y^3}{x^3-y^3} = \frac{125+27}{125-27} \quad [\text{On applying componendo and dividendo}]$$

$$= \frac{152}{98} = \frac{76}{49} \quad \text{Ans.}$$

Alternative method :

$$x : y = 5 : 3 \Rightarrow \text{if } x = 5k \text{ then } y = 3k$$

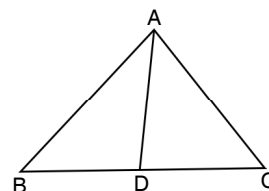
$$\therefore \frac{x^3+y^3}{x^3-y^3} = \frac{125k^3+27k^3}{125k^3-27k^3} = \frac{152k^3}{98k^3} = \frac{76}{49} \quad \text{Ans.}$$

(c) In $\triangle ABC$, $\angle ABC = \angle DAC$, $AB = 8$ cm, $AC = 4$ cm and $AD = 5$ cm.

(i) Prove that $\triangle ACD$ is similar to $\triangle BCA$.

(ii) Find BC and CD .

(iii) Find area of $\triangle ACD$: area of $\triangle ABC$. [4]



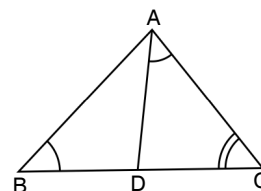
Solution :

(i) In $\triangle ACD$ and $\triangle BCA$

$$\angle ABC = \angle DAC \quad (\text{given})$$

$$\angle C = \angle C \quad (\text{common})$$

$$\therefore \triangle ACD \sim \triangle BCA \quad (\text{by A.A.})$$

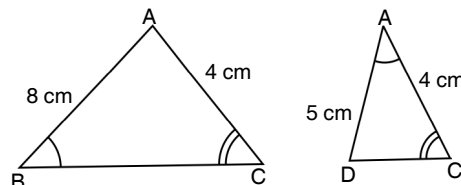


(ii) $\triangle ACD \sim \triangle BCA$

$$\Rightarrow \frac{BC}{AC} = \frac{AB}{AD} = \frac{AC}{CD}$$

$$\Rightarrow \frac{BC}{4} = \frac{8}{5} = \frac{4}{CD}$$

$$\Rightarrow BC = \frac{8}{5} \times 4 \text{ cm} = 6.4 \text{ cm} \quad \text{and} \quad CD = \frac{5 \times 4}{8} \text{ cm} = 2.5 \text{ cm} \quad \text{Ans.}$$



Question 6 :

(a) Find the value of a for which the points $A(a, 3)$, $B(2, 1)$ and $C(5, a)$ are collinear. Hence, find the equation of the line. [3]

Solution :

A, B and C are collinear

$$\Rightarrow \text{slope of AB} = \text{slope of BC}$$

$$\Rightarrow \frac{1-3}{2-a} = \frac{a-1}{5-2} \quad \text{i.e. } 2a - 2 - a^2 + a = -6$$

$$\Rightarrow a^2 - 3a - 4 = 0$$

$$\Rightarrow a = 4 \quad \text{or} \quad a = -1$$

$$a = 4 \Rightarrow A = (4, 3) \text{ and } B = (2, 1)$$

$$\therefore \text{Slope of AB} = \frac{1-3}{2-4} = 1$$

And, equation of the line is :

$$y - 3 = 1(x - 4) \quad [y - y_1 = m(x - x_1)]$$

$$\Rightarrow y - 3 = x - 4$$

$$\Rightarrow x - y = 1 \quad \text{Ans.}$$

$$a = -1 \Rightarrow A = (-1, 3) \text{ and } B = (2, 1)$$

$$\therefore \text{Slope of AB} = \frac{1-3}{2+1} = \frac{-2}{3}$$

Equation of the line is :

$$y - 3 = \frac{-2}{3}(x + 1)$$

$$\Rightarrow 3y - 9 = -2x - 2$$

$$\Rightarrow 2x + 3y = 7 \quad \text{Ans.}$$

(b) Salman invests a sum of money in ₹ 50 shares, paying 15% dividend quoted at 20% premium. If his annual dividend is ₹ 600, calculate :

(i) the number of shares he bought.

(ii) his total investment.

(iii) the rate of return on his investment. [3]

Solution :

Given : N.V. of each share = ₹ 50

M.V. of each share = ₹ 50 + 20% of ₹ 50 = ₹ 60

Dividend on one share = 15% of ₹ 50 = ₹ 7.50

$$(i) \quad \text{Number of shares bought} = \frac{\text{Total dividend}}{\text{Dividend on one share}} = \frac{\text{₹ } 600}{\text{₹ } 7.50} = 80 \quad \text{Ans.}$$

$$(ii) \quad \text{His total investment} = \text{Number of shares} \times \text{M.V. of one share} \\ = 80 \times \text{₹ } 60 = \text{₹ } 4,800 \quad \text{Ans.}$$

$$(iii) \quad \text{Rate of return} = \frac{\text{Total dividend}}{\text{Total investment}} \times 100\% \\ = \frac{\text{₹ } 600}{\text{₹ } 4,800} \times 100\% = 12.5\% \quad \text{Ans.}$$

Alternative method :

Rate of return $\times$ M.V. = Rate of dividend $\times$ N.V.

$$\Rightarrow \text{Rate of return} \times 60 = 15\% \times 50$$

$$\Rightarrow \text{Rate of return} = \frac{15 \times 50}{60} \% = 12.5\% \quad \text{Ans.}$$

(c) The surface area of a solid metallic sphere is 2464 cm^2 . It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm . Calculate :

(i) the radius of the sphere.

(ii) the number of cones recast. (Take $\pi = \frac{22}{7}$) [4]

Solution :

$$(i) \quad 4\pi r^2 = 2464 \Rightarrow r^2 = \frac{2464 \times 7}{4 \times 22} = 196$$

$$\Rightarrow \text{Radius } (r) = 14 \text{ cm} \quad \text{Ans.}$$

$$(ii) \quad \therefore \text{Volume of sphere melted} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \times (14)^3 \text{ cm}^3$$

$$\text{and, volume of each cone recasted} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times (3.5)^2 \times 7 \text{ cm}^3$$

$$\therefore \text{The number of cones recasted} = \frac{\text{Volume of sphere melted}}{\text{Volume of each cone formed}}$$

$$= \frac{\frac{4}{3}\pi \times 14 \times 14 \times 14}{\frac{1}{3}\pi \times 3.5 \times 3.5 \times 7} = 128 \quad \text{Ans.}$$

Question 7 :

(a) Calculate the mean of the distribution, given below using the short cut method.

Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80
No. of students	2	6	10	12	9	7	4

Solution : [3]

Marks	f	Class-mark (x)	$d = x - A$	$f \times d$
11-20	2	15.5	-30	-60
21-30	6	25.5	-20	-120
31-40	10	35.5	-10	-100
41-50	12	45.5 = A	0	0
51-60	9	55.5	10	90
61-70	7	65.5	20	140
71-80	4	75.5	30	120

$$\Sigma f = 50$$

$$\Sigma f \times d = 70$$

$$\therefore \text{Mean} = A + \frac{\Sigma fd}{\Sigma f}$$

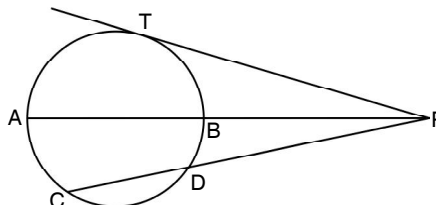
$$= 45.5 + \frac{70}{50}$$

$$= 45.5 + 1.4$$

$$= 46.9 \quad \text{Ans.}$$

(b) In the given figure, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. $CD = 7.8 \text{ cm}$, $PD = 5 \text{ cm}$, $PB = 4 \text{ cm}$. Find :

(i) AB. (ii) the length of tangent PT. [3]



Solution :

- (i) Let $AB = x$ cm
 $\Rightarrow PA = PB + x = (4 + x)$ cm
 $PC = PD + CD = (5 + 7.8)$ cm = 12.8 cm
and $PD = 5$ cm
Since, $PA \times PB = PC \times PD$
 $\Rightarrow (4 + x) \times 4 = 12.8 \times 5 \Rightarrow x = 12$ i.e. **$AB = 12$ cm** **Ans.**
- (ii) Since, $PT^2 = PC \times PD$
 $\Rightarrow PT^2 = 12.8 \times 5 = 64$ i.e. **$PT = 8$ cm** **Ans.**

(c) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$. Find $A^2 + AC - 5B$. [4]

Solution :

$$\begin{aligned} \therefore A^2 &= A \times A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\ AC &= \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} \\ \text{and, } 5B &= 5 \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix} \\ \therefore A^2 + AC - 5B &= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix} \\ &= \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix} = \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix} \quad \text{Ans.} \end{aligned}$$

Question 8 :

- (a) The compound interest, calculated yearly, on a certain sum of money for the second year is ₹ 1,320 and for the third year is ₹ 1,452. Calculate the rate of interest and the original sum of money. [3]

Solution :

$$\begin{aligned} \text{C.I. for 2nd year} &= ₹ 1,320; \\ \text{C.I. for 3rd year} &= ₹ 1,452 \\ \text{and, } ₹ 1,452 - ₹ 1,320 &= ₹ 132 \\ \Rightarrow \text{Interest of one year on } ₹ 1,320 &= ₹ 132 \\ \therefore \text{Rate of interest} &= \frac{₹ 132}{₹ 1,320} \times 100\% = 10\% \quad \text{Ans.} \end{aligned}$$

Alternative method :

$$\begin{aligned} \text{Let rate of interest be } x\% \\ \therefore \text{C.I. of 2nd year} + \text{Interest on it for 1 year} &= \text{C.I. of 3rd year} \\ \Rightarrow ₹ 1,320 + x\% \text{ of } ₹ 1,320 &= ₹ 1,452 \\ \Rightarrow x = 10 \text{ i.e. Rate of interest} &= 10\% \quad \text{Ans.} \end{aligned}$$

Let the original sum = ₹ P

$$\begin{aligned} \text{For 1st year : Interest} &= \frac{₹ P \times 10 \times 1}{100} = \frac{₹ P}{10} \\ \text{Amount} &= P + I = ₹ P + \frac{₹ P}{10} = \frac{₹ 11P}{10} \end{aligned}$$

For 2nd years :
$$I = \frac{\text{₹}11P}{10} \times \frac{10 \times 1}{100} = \frac{\text{₹}11P}{100}$$

Given C.I. of 2nd year = ₹ 1,320 $\Rightarrow \frac{\text{₹}11P}{100} = \text{₹} 1,320 \Rightarrow P = 12,000$

i.e. **Original sum = ₹ 12,000**

Ans.

- (b) Construct a ΔABC with $BC = 6.5$ cm, $AB = 5.5$ cm, $AC = 5$ cm. Construct the incircle of the triangle. Measure and record the radius of the incircle. [3]

Solution :

Steps : 1. Draw $BC = 6.5$ cm.

2. With B as centre, draw an arc of radius 5.5 cm (= AB) and with C as centre, draw one more arc of radius 5 cm (= AC). Let these two arcs intersect each other at point A.

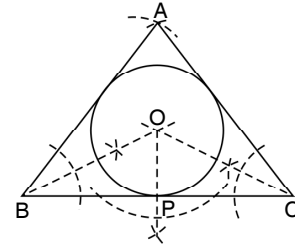
3. Complete the triangle ABC.

4. Draw bisectors of angles ABC and ACB, which meet each other at point O.

5. Through point O, draw $OP \perp BC$.

6. With O as centre and OP as radius, draw the required incircle which will touch all the three sides of the triangle.

7. Measure OP, the radius of the incircle.



Radius of the incircle = 1.6 cm (app.)

Ans.

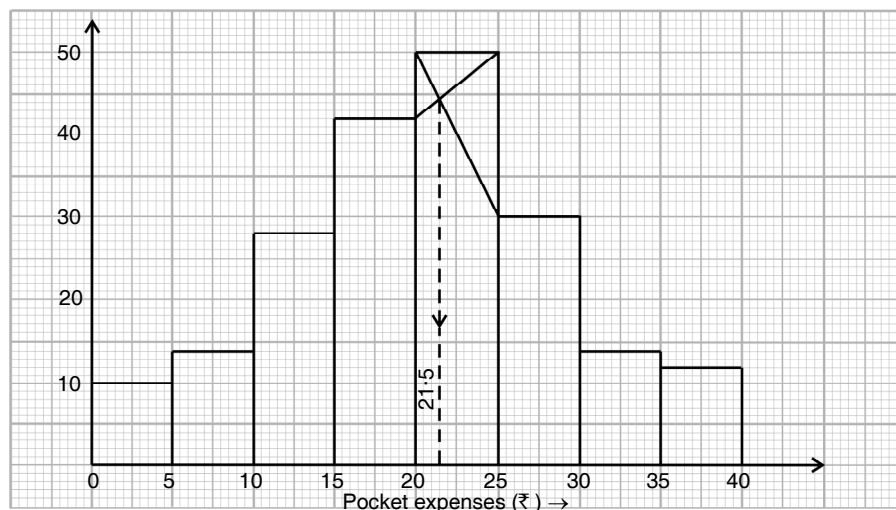
- (c) (Use a graph paper for this question). The daily pocket expenses of 200 students in a school are given below :

Pocket expenses (in ₹)	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students (frequency)	10	14	28	42	50	30	14	12

Draw a histogram representing the above distribution and estimate the mode from the graph. [4]

Solution :

The required histogram is as drawn below :



Clearly, **mode = 21.5**

Ans.

Question 9 :

(a) If $(x - 9) : (3x + 6)$ is the duplicate ratio of $4 : 9$, find the value of x . [3]

Solution :

$$\text{Given : } \frac{x-9}{3x+6} = \frac{4^2}{9^2} = \frac{16}{81}$$

$$\Rightarrow 81x - 729 = 48x + 96$$

$$\Rightarrow 33x = 825 \text{ i.e. } x = 25 \quad \text{Ans.}$$

(b) Solve for x using the quadratic formula. Write your answer correct to two significant figures. $(x - 1)^2 - 3x + 4 = 0$. [3]

Solution :

$$\Rightarrow x^2 - 2x + 1 - 3x + 4 = 0$$

$$\Rightarrow x^2 - 5x + 5 = 0$$

Comparing with $ax^2 + bx + c = 0$; we get :

$$a = 1, b = -5 \text{ and } c = 5$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{5 \pm \sqrt{5}}{2} = \frac{5 \pm 2.236}{2}$$

$$= \frac{7.236}{2} \text{ or } \frac{2.764}{2}$$

$$= 3.618 \text{ or } 1.382$$

$$\therefore x \text{ (correct to two significant figures)} = 3.6 \text{ or } 1.4 \quad \text{Ans.}$$

(c) A page from the saving bank account of Priyanka is given below :

Date	Particulars	Amount withdrawn (₹)	Amount deposited (₹)	Balance (₹)
03/04/2006	B/F			4000.00
05/04/2006	By cash		2000.00	6000.00
18/04/2006	By cheque		6000.00	12000.00
25/05/2006	To cheque	5000.00		7000.00
30/05/2006	By cash		3000.00	10000.00
20/07/2006	By self	4000.00		6000.00
10/09/2006	By cash		2000.00	8000.00
19/09/2006	To cheque	1000.00		7000.00

If the interest earned by Priyanka for the period ending September, 2006 is ₹ 175, find the rate of interest. [4]

Solution :

Principal for April = ₹ 6,000

Principal for May = ₹ 7,000

Principal for June = ₹ 10,000

Principal for July = ₹ 6,000
 Principal for August = ₹ 6,000
 Principal for September = ₹ 7,000

 Equivalent principal for 1 month = ₹ 42,000

$$\therefore \text{Rate of interest} = \frac{I \times 100}{P \times T} \% = \frac{175 \times 100 \times 12}{42000 \times 1} \% = 5\% \quad \text{Ans.}$$

Question 10 :

(a) A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. [4]

Solution :

Let two digit number be $10x + y$

Given : $xy = 6$ and $10x + y + 9 = 10y + x$

$$\Rightarrow y = \frac{6}{x} \text{ and } 9x - 9y = -9 \text{ i.e. } x - y = -1$$

$$\text{Now, } x - y = -1 \Rightarrow x - \frac{6}{x} = -1 \text{ i.e. } x^2 + x - 6 = 0$$

$$\Rightarrow (x + 3)(x - 2) = 0 \text{ i.e. } x = -3 \text{ or } x = 2$$

$$\text{Rejecting } x = -3 ; \text{ we get : } x = 2 \text{ and } y = \frac{6}{x} \Rightarrow y = \frac{6}{2} = 3$$

$$\therefore \text{Number} = 10x + y = 10 \times 2 + 3 = 23 \quad \text{Ans.}$$

(b) The marks obtained by 100 students in a mathematics test are given below :

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	7	12	17	23	14	9	6	5	4

Draw an ogive for the given distribution on a graph sheet.

Use a scale of 2 cm = 10 units on both the axis.

Use the ogive to estimate line :

- median.
- lower quartile.
- number of students who obtained more than 85% marks in the test.
- number of students failed, if the pass percentage was 35. [6]

Solution :

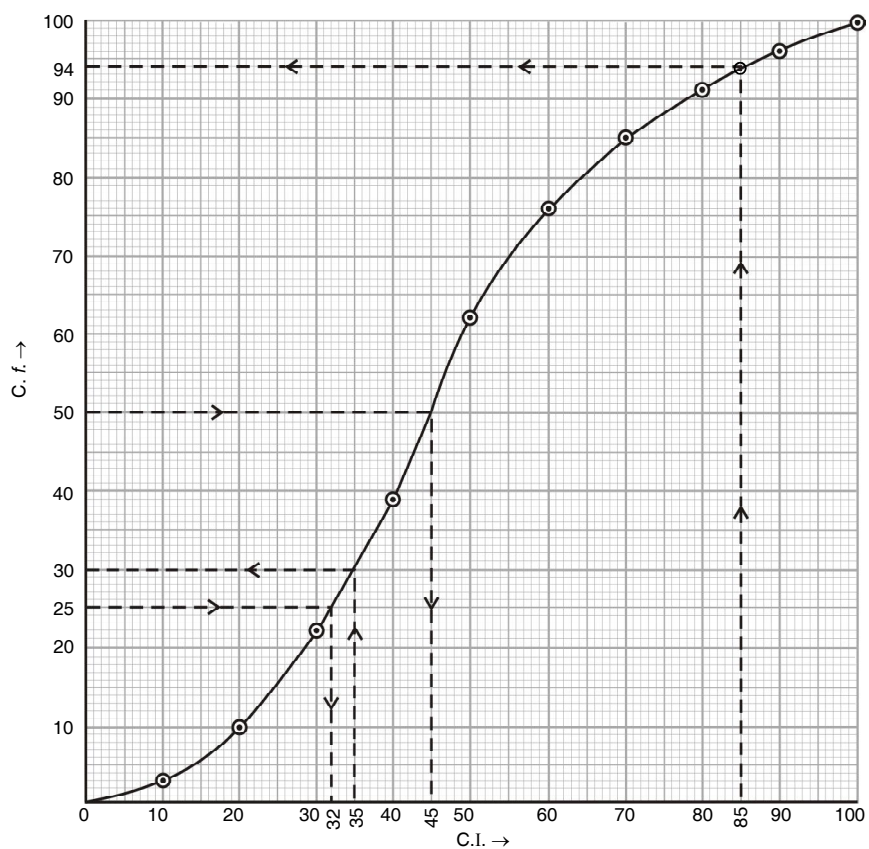
C.I.	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
f	3	7	12	17	23	14	9	6	5	4
$C.f.$	3	10	22	39	62	76	85	91	96	100

Taking 2 cm = 10 units on both the axis; plot the points (10, 3), (20, 10), (30, 22), (40, 39), (50, 62), (60, 76), (70, 85), (80, 91), (90, 96) and (100, 100) on a graph paper.

Then draw a free-hand curve (ogive) passing through all the points plotted.

[The graph must start from the lower limit of first class and terminate at upper limit of last class].

The resulting graph (ogive) will be as shown below.



(i) **Median** = $\left(\frac{100}{2}\right)^{\text{th}}$ term = 50th term = **45** **Ans.**

(ii) **Lower quartile** = $\left(\frac{100}{4}\right)^{\text{th}}$ term = 25th term = **32** **Ans.**

(iii) Since, number of students getting upto 85% = 94

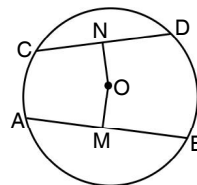
$\therefore$ **Number of students who obtained more than 85% marks in the class**
 $= 100 - 94 = 6$ **Ans.**

(iv) **If the pass percent was 35, number of students who failed = 30.** **Ans.**

Question 11 :

(a) In the given figure, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD. AB = 24 cm, OM = 5 cm, ON = 12 cm. Find the :

- (i) radius of the circle.
- (ii) length of chord CD.



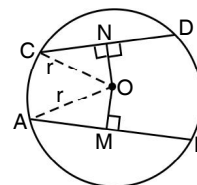
[3]

Solution :

(i) In $\triangle OAM$,

$$AM = \frac{1}{2} AB = 12 \text{ cm}$$

$$OM = 5 \text{ cm}$$



$$\begin{aligned}\text{and, } OA^2 &= AM^2 + OM^2 \\ &= 12^2 + 5^2 = 144 + 25 = 169 \quad i.e. \quad OA = 13\end{aligned}$$

$\therefore$ **Radius of the circle** = OA = 13 cm

Ans.

(ii) In $\triangle ONC$,

OC = radius = 13 cm, ON = 12 cm

and, $OC^2 = CN^2 + ON^2$

$$\Rightarrow 13^2 = CN^2 + 12^2 \Rightarrow CN^2 = 169 - 144 = 25 \quad i.e. \quad CN = 5$$

$\therefore$ **Length of chord CD** = $2 \times CN = 2 \times 5 \text{ cm} = 10 \text{ cm}$

Ans.

(b) Prove the identity $(\sin \theta + \cos \theta)(\tan \theta + \cot \theta) = \sec \theta + \operatorname{cosec} \theta$. [3]

Solution :

$$\begin{aligned}\text{LHS.} &= (\sin \theta + \cos \theta) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) \\ &= (\sin \theta + \cos \theta) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) \\ &= (\sin \theta + \cos \theta) \left(\frac{1}{\cos \theta \sin \theta} \right) \\ &= \frac{\sin \theta}{\cos \theta \sin \theta} + \frac{\cos \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta} + \frac{1}{\sin \theta} = \sec \theta + \operatorname{cosec} \theta = \text{R.H.S}\end{aligned}$$

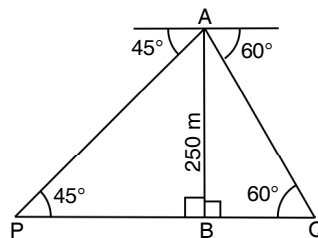
(c) An aeroplane, at an altitude of 250 m, observes the angles of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number. [4]

Solution :

According to the given statement, the figure will be as shown alongside, in which A is the position of the aeroplane so that AB = 250 m.

P and Q be two boats on the opposite banks of the river with angles of depression 45° and 60° from A.

To find : Length of PQ.



$$\begin{aligned}\text{In } \triangle APB, \quad \tan 45^\circ &= \frac{AB}{PB} \\ \Rightarrow \quad 1 &= \frac{250 \text{ m}}{PB} \quad i.e. \quad PB = 250 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{In } \triangle AQB, \quad \tan 60^\circ &= \frac{AB}{BQ} \\ \Rightarrow \quad \sqrt{3} &= \frac{250 \text{ m}}{BQ} \quad i.e. \quad BQ = \frac{250}{\sqrt{3}} \text{ m}\end{aligned}$$

$$BQ = \frac{250 \text{ m}}{\sqrt{3}} \text{ m} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{250 \times 1.732}{3} \text{ m} = 144.3 \text{ m}$$

$$\begin{aligned}\therefore \quad \text{Length of PQ} &= PB + BQ \\ &= 250 \text{ m} + 144.3 \text{ m} = 394.3 \text{ m} \\ &= \mathbf{394 \text{ m}} \text{ (to the nearest whole number)}\end{aligned}$$

Ans.

